Problem 1

Given the system

$$\dot{\mathbf{x}} = \mathbf{A}\mathbf{x} + \mathbf{B}u, \qquad y = \mathbf{C}\mathbf{x}, \tag{1}$$

with matrices

$$\mathbf{A} = \begin{bmatrix} -7 & -12 \\ 1 & 0 \end{bmatrix}, \qquad \mathbf{B} = \begin{bmatrix} 1 \\ 0 \end{bmatrix}, \qquad \mathbf{C} = \begin{bmatrix} 1 & 2 \end{bmatrix}.$$

a) Find a controller matrix $\mathbf{K}$ such that the poles of the controlled system (1) are at $s = -1 \pm j$.

b) Do the system zeroes of (1) change by state feedback?

Problem 2

For the system

$$\dot{\mathbf{x}} = \begin{pmatrix} 0 & 1 \\ -6 & -5 \end{pmatrix} \mathbf{x} + \begin{pmatrix} 0 \\ 1 \end{pmatrix} u$$

$$y = \begin{pmatrix} 1 & 0 \end{pmatrix} \mathbf{x}$$

design a state feedback controller that satisfies the following specifications:

- Closed-loop poles have a damping coefficient of $\zeta = 0.707$.
- Step-response peak time is not greater than 3.14 sec.

Hint: System peak time is given by

$$t_p = \frac{\pi}{\sqrt{1 - \zeta^2 \omega_n}}$$

Problem 3

Given the plant

$$\dot{\mathbf{x}} = \mathbf{A}\mathbf{x} + \mathbf{B}u, \qquad y = \mathbf{C}\mathbf{x},$$

with matrices

$$\mathbf{A} = \begin{bmatrix} 0 & 1 \\ 0 & -10 \end{bmatrix}, \qquad \mathbf{B} = \begin{bmatrix} 0 \\ 1 \end{bmatrix}, \qquad \mathbf{C} = \begin{bmatrix} 1 & 0 \end{bmatrix}.$$

Find a state feedback controller matrix $\mathbf{K}$ such that the closed-loop step response has an overshoot of less than $M_p \leq 25\%$ and a $S_t = 1\%$ settling time under 0.115 sec.

Hint: The overshoot specification is $M_p = e^{\frac{-\pi\zeta}{\sqrt{1-\zeta^2}}}$,
and the settling time specification is $S_t = e^{-\zeta\omega_n t_s}$.

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